

Torsion and rigidity modulus.
[Continued]

$$\therefore F = \eta x \theta$$

Now this face area of this hollow cylinder
 $= 2\pi x dx$

\therefore Total shearing force on this area

$$= 2\pi x dx \times \frac{\eta x \theta}{l}$$

$$= \frac{2\pi \eta \theta}{l} x^2 dx$$

Therefore, moment of this force about the axis OO' is equal to

$$= \frac{2\pi \eta \theta}{l} x^2 dx \times x$$

$$= \frac{2\pi \eta \theta}{l} x^3 dx$$

Integrating this expression between the limits $x=0$ to $x=r$, we have

Total twisting couple on the cylinder

$$= \int_0^r \frac{2\pi \eta \theta}{l} x^3 dx$$

$$= \frac{2\pi \eta \theta}{l} \int_0^r x^3 dx$$

$$= \frac{2\pi \eta \theta}{l} \left[\frac{x^4}{4} \right]_0^r$$

$$= \frac{2\pi \eta \theta}{4l} r^4$$

$$= \frac{\pi \eta \theta r^4}{2l}$$

$$= \frac{\pi \eta \theta r^4}{2l}$$

If $\theta = 1$ radian, we have twisting couple per unit twist of the cylinder or wire

$$C = \frac{\pi \eta r^4}{2l}$$

This twisting couple, per unit twist of the wire is also called the torsional Rigidity.